1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.
2. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?

Answer:

1. Let's consider the probability that at least one of the seasons is missing among the 7 birthdays. We can calculate this using the principle of inclusion-exclusion as follows:

P(at least one season missing) = P(winter missing) + P(spring missing) + P(summer missing) + P(fall missing) - P(winter and spring missing) - P(winter and summer missing) - P(winter and fall missing) - P(spring and summer missing) - P(spring and fall missing) - P(summer and fall missing) + P(winter, spring, and summer missing) + P(winter, spring, and fall missing) + P(winter, summer, and fall missing) + P(spring, summer, and fall missing) - P(winter, spring, summer, and fall missing)

Each of the individual probabilities can be calculated using the multiplication rule as follows:

P(winter missing) = (3/4)^7 P(winter and spring missing) = (2/4)^7 P(winter, spring, and summer missing) = (1/4)^7

The other probabilities can be calculated similarly. The last term in the formula above corresponds to the probability that all 7 birthdays occur in only 3 seasons, which is 0 since there are only 4 seasons.

Using a calculator or computer, we can compute each of these probabilities and substitute them into the formula above to obtain:

P(all 4 seasons occur at least once) = 1 - P(at least one season missing) = 1 - (15/16)^7 + (6/16)^7 - (1/16)^7 ≈ 0.866

Therefore, the probability that all 4 seasons occur at least once among the 7 birthdays is approximately 0.866.

1. There are a total of 6^5 = 7776 ways for Alice to choose 5 classes for each day of the week, since she has 6 choices for each day. If she wants to have classes every day, then she must choose 5 classes for Monday through Friday and 2 additional classes for any day. There are 6 choices for each of these 2 additional classes. Therefore, the total number of ways for Alice to choose 7 classes with at least one class for each day is:

N = 6^5 × 6^2 = 6^7 = 279936

Since Alice is equally likely to choose any of these 7-class combinations, the probability that she chooses one with at least one class for each day is:

P(at least one class for each day) = N / (total number of 7-class combinations)

To find the total number of 7-class combinations, we can use the binomial coefficient as follows:

(total number of 7-class combinations) = (30 choose 7) = 2035800

Substituting this into the formula above, we obtain:

P(at least one class for each day) = 279936 / 2035800 ≈ 0.1374

Therefore, the probability that Alice will have classes every day, Monday through Friday, is approximately 0.1374.